Next-to-Leading Order Tools for Colliders

2004 CTEQ Summer School

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Argonne National Laboratory

Lecture outline

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Theory overview

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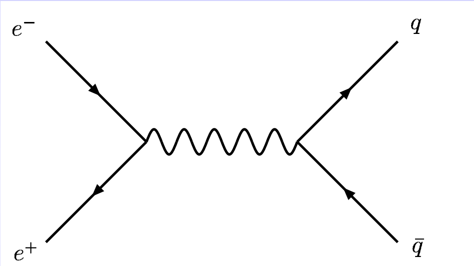

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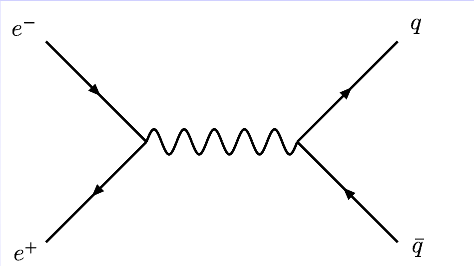

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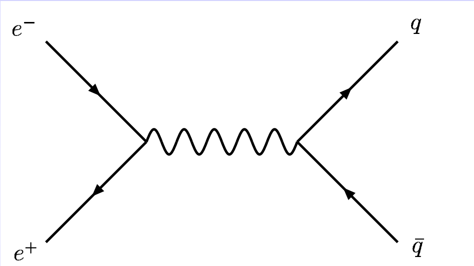

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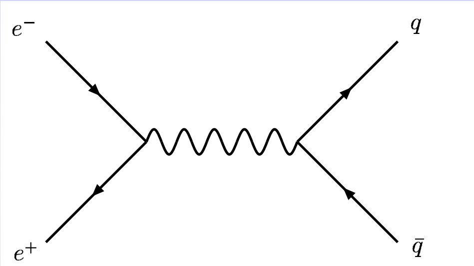

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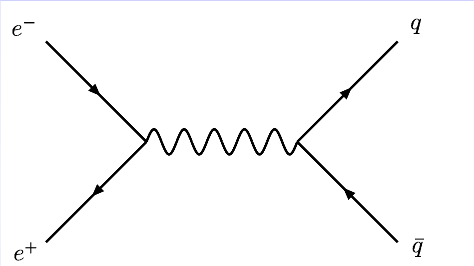

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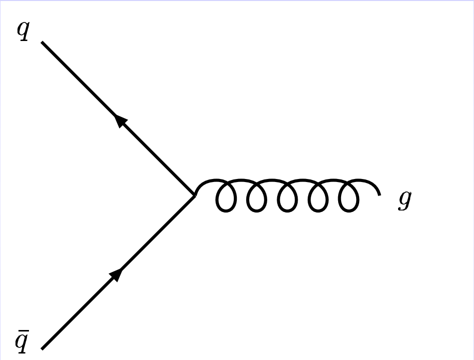
Gamma-matrix warmup exercise: show that the spin- and colour-summed squared matrix element is given by $\sum |\mathcal{M}|^2 = 8Ne^4Q^2 \left(\frac{(p_1.q_1)^2+(p_1.q_2)^2}{(p_1.p_2)^2}\right)$, where $Q =$ quark charge.

Adding QCD

- To calculate the effect of NLO QCD, we have to add contributions which are proportional to α_s. In other words, we need a total of two extra couplings of quarks to a gluon:



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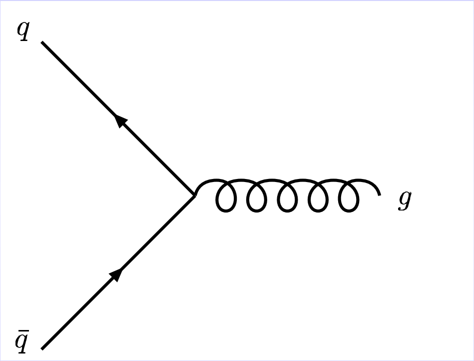

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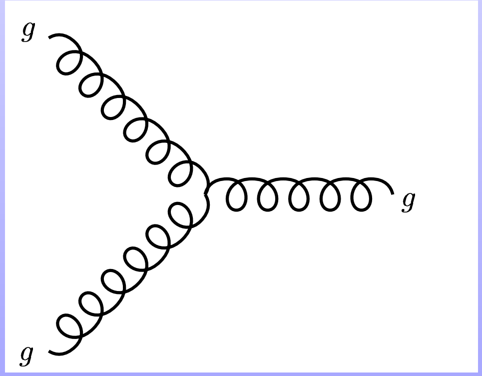
- In general, we can attach gluons in more complicated ways



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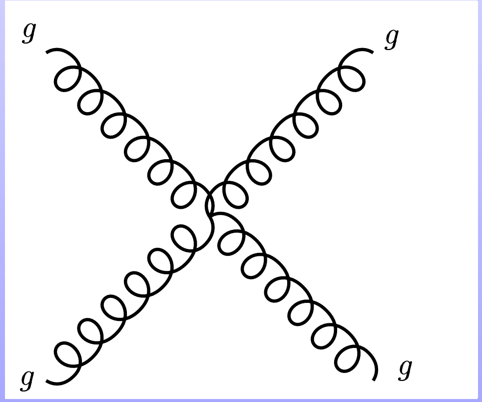

$\sim \quad g_s^2$

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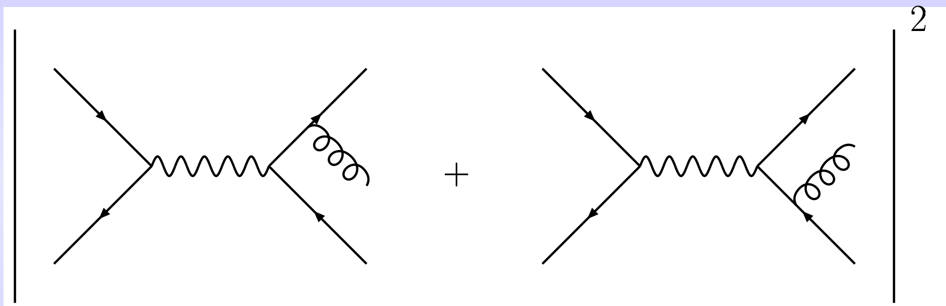

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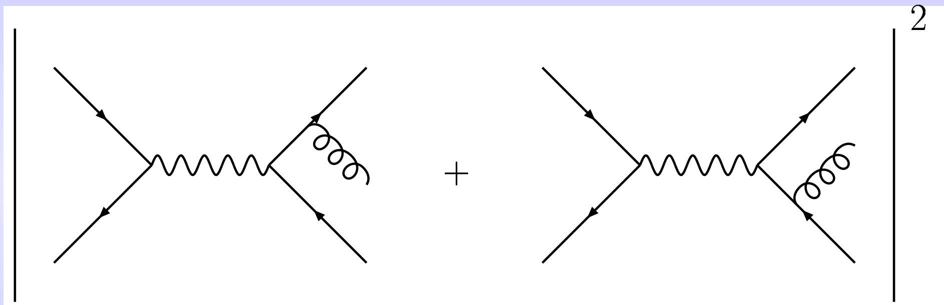

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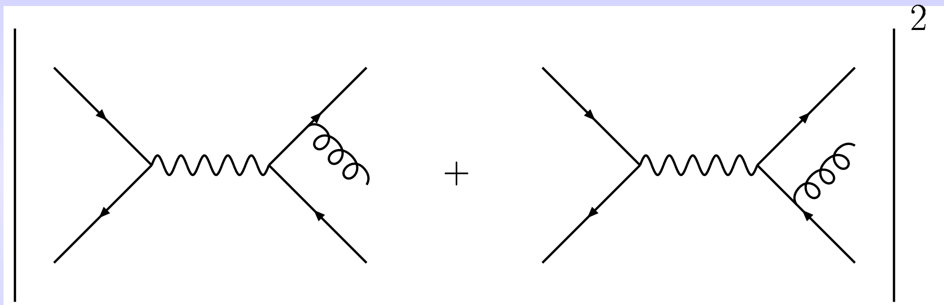

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- There's another set of diagrams to consider though ...

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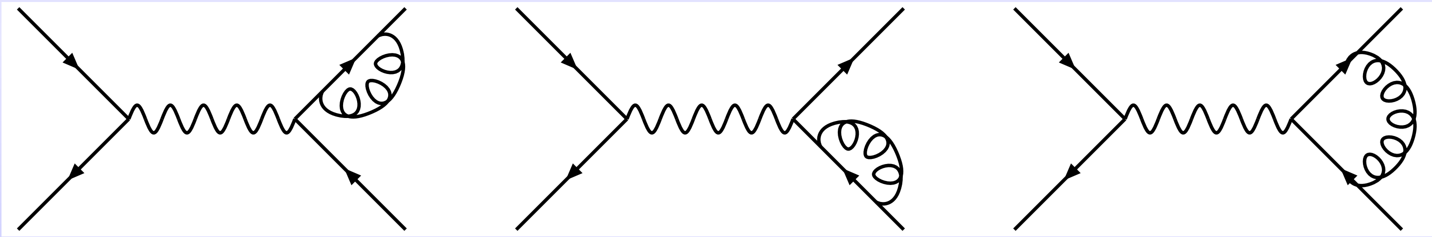

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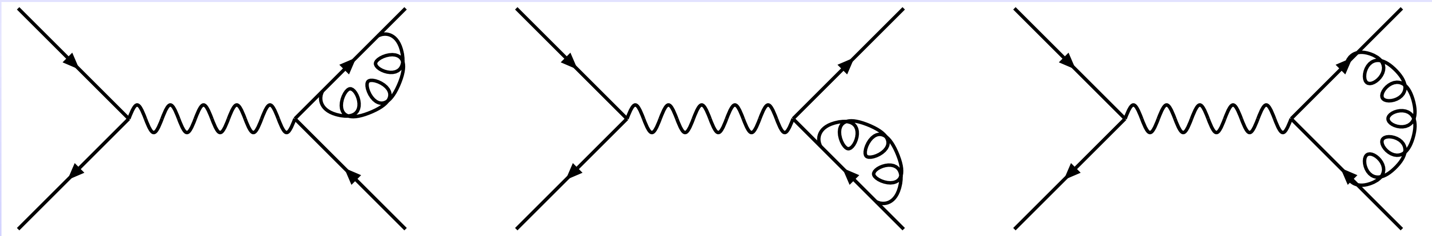

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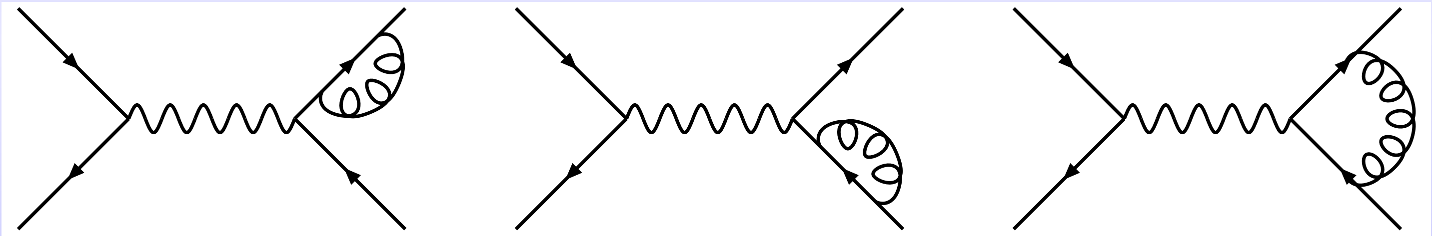

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- One calculates the NLO cross-section by summing the real and virtual contributions

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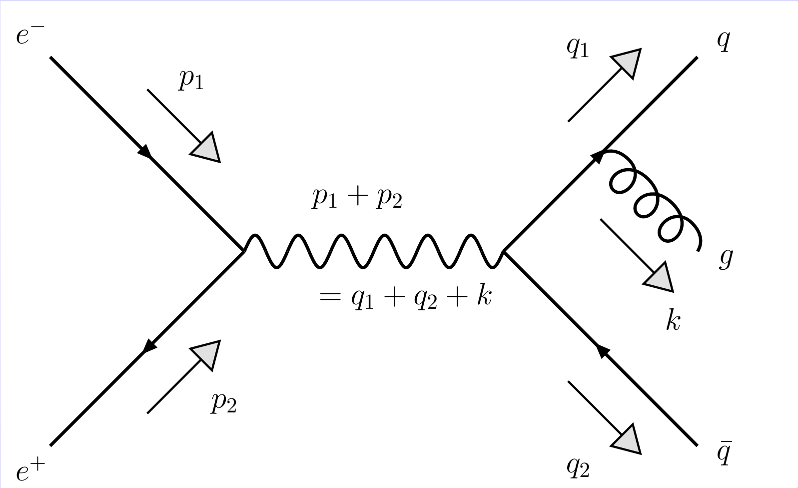

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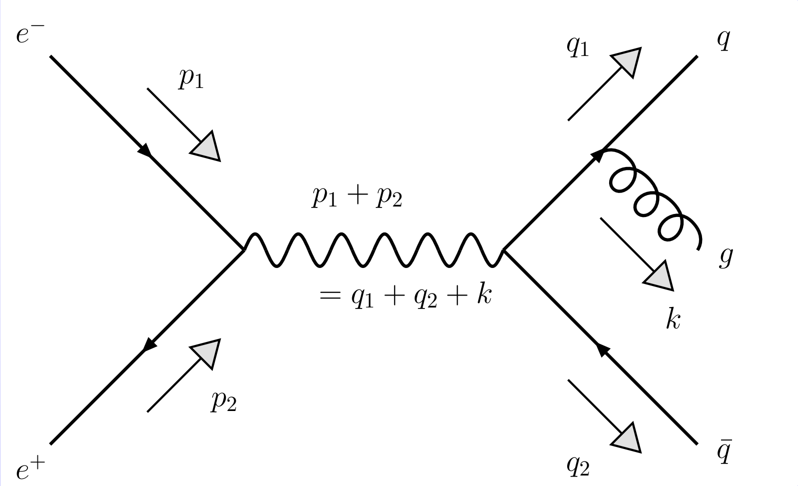

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- The intermediate quark propagator before the gluon emission contributes a factor of

$$\frac{1}{(q_1 + k)^2} = \frac{1}{2q_1.k}, \qquad \text{since } q_1^2 = k^2 = 0.$$

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  where θ_{qg}($\theta_{\bar{q}g}$) is the angle between the gluon and the (anti-)quark.
- These propagators can clearly vanish in a number of cases

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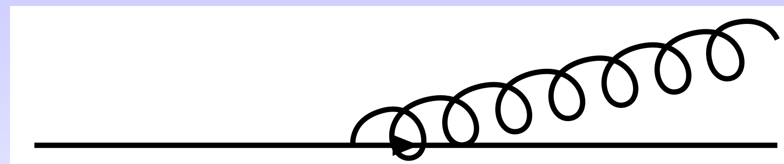

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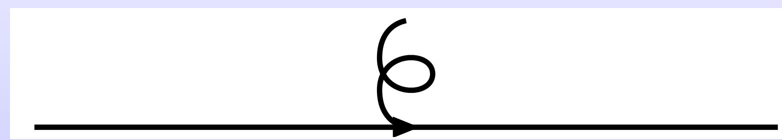

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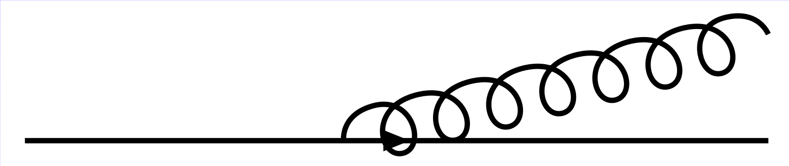

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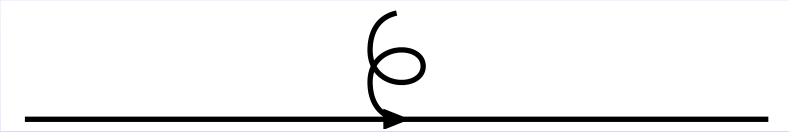

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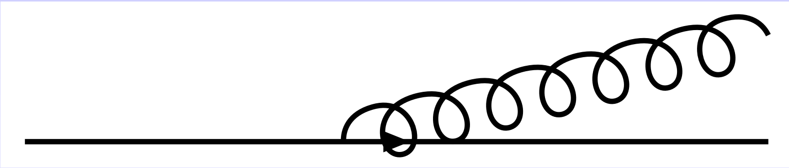

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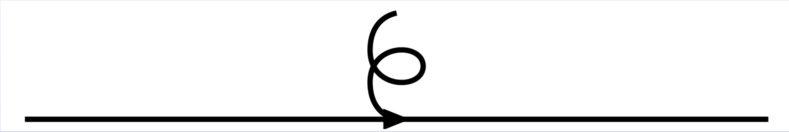

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- Together, these two problems are called infrared singularities

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- The most common trick for proceeding is to use dimensional regularization (DR):

$$D = 4 \qquad \longrightarrow \qquad D = 4 - 2\epsilon$$

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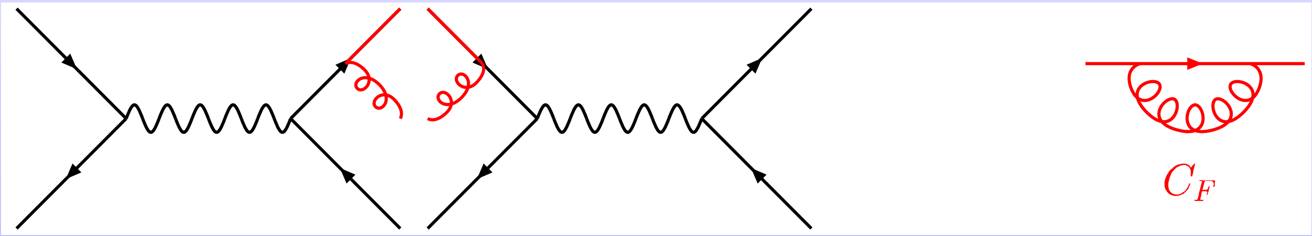

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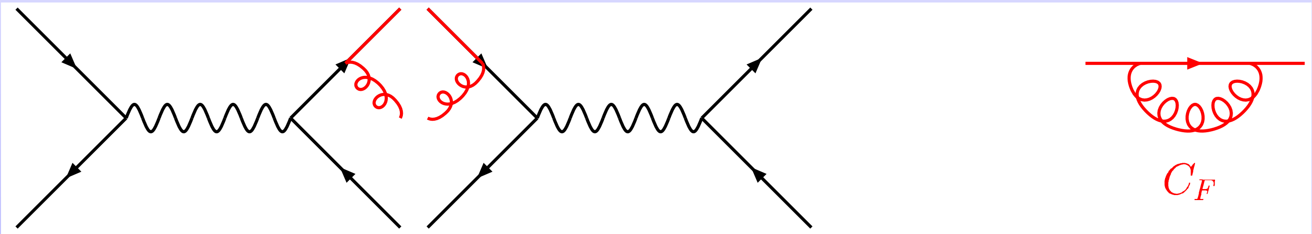

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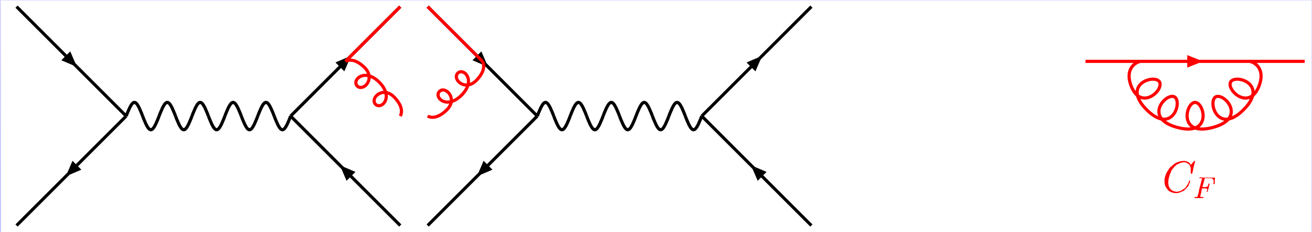

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- Notice that, in particular, the ϵ poles are proportional to the lowest order result. This is a crucial observation - more on this later.

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$$\frac{d^4\ell \quad \mathcal{N}}{\ell^2(\ell+q_2)^2(\ell+q_1+q_2)^2}$$

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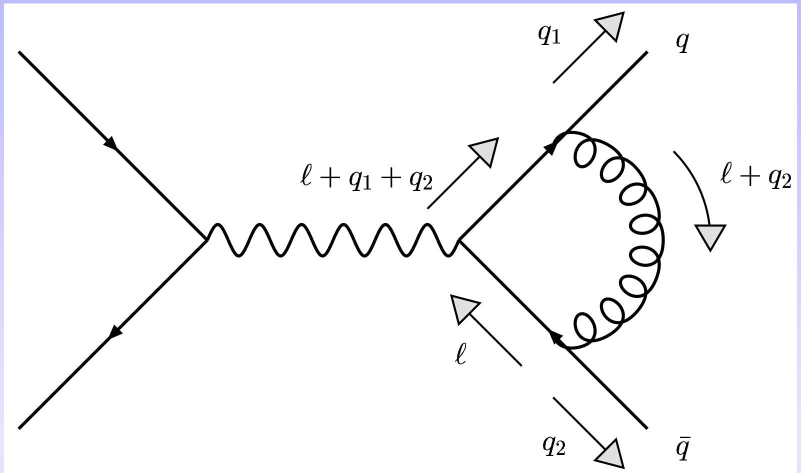

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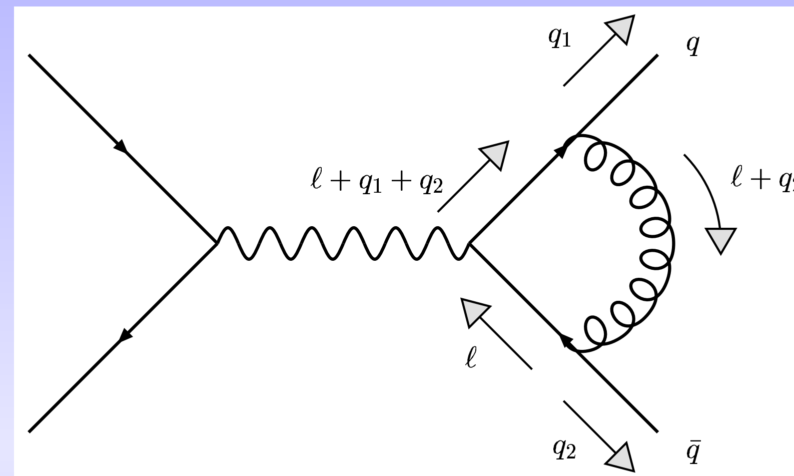

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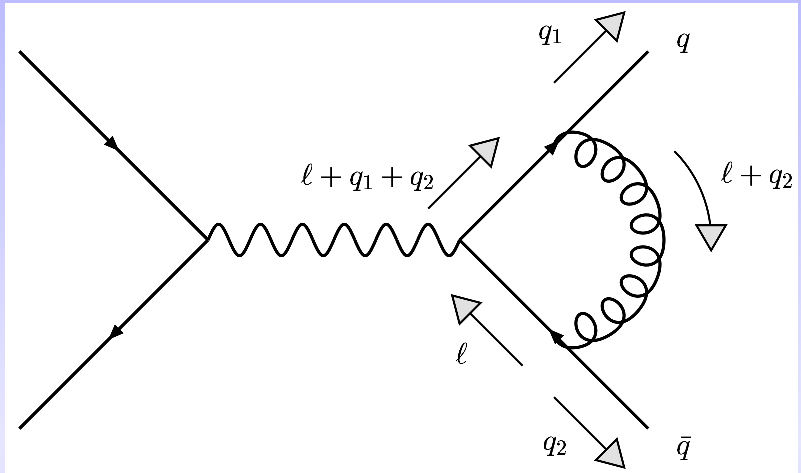

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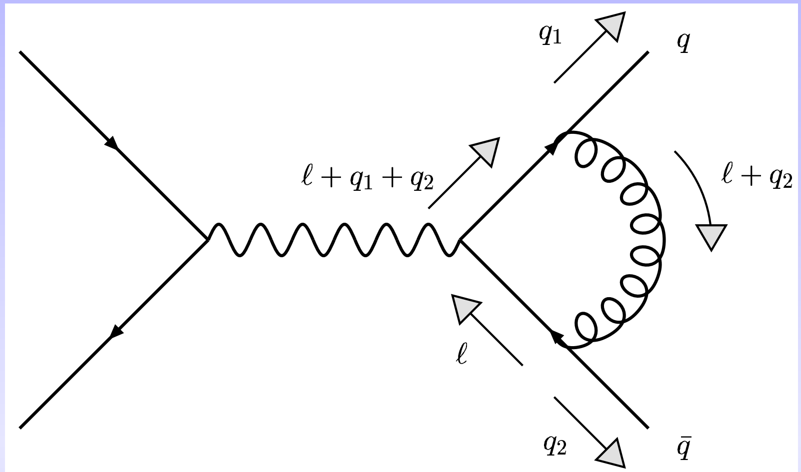

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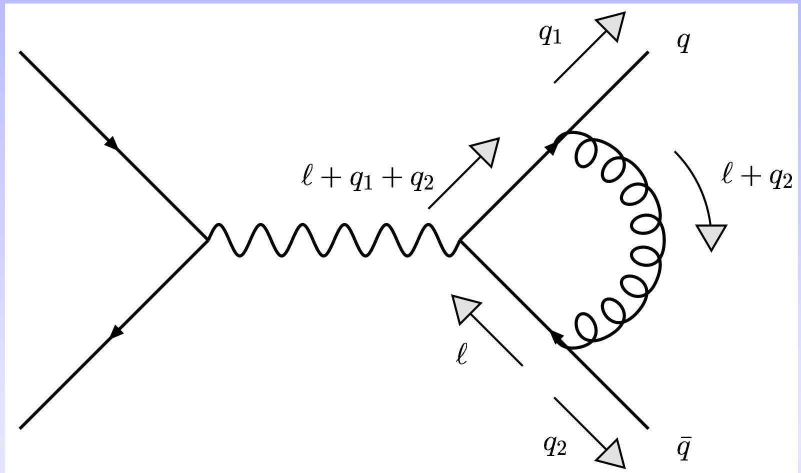

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  - ⋆ In fact, $\ell = xq_2$ for any value of x also makes two propagators vanish - another collinear singularity.
- Moreover, as $|\ell| \to \infty$, power counting makes some terms look logarithmically divergent, $\sim \int \frac{dy}{y}$ (ultraviolet divergence)

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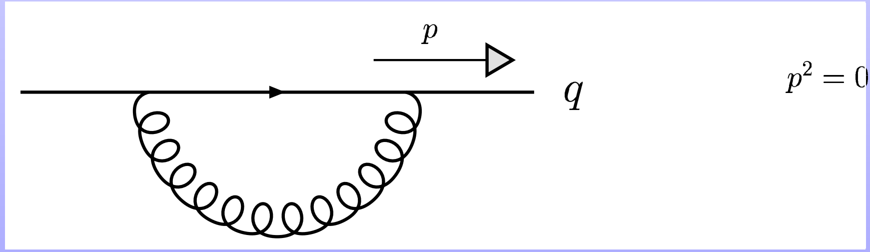
- Note that DR makes the contribution from our bubble diagrams vanish. For this reason, sometimes the diagrams for self-energy corrections on massless external quarks are not even written down



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- A NLO Monte Carlo is so-called because of the integration technique that is used to evaluate the phase space integral over the appropriate matrix elements

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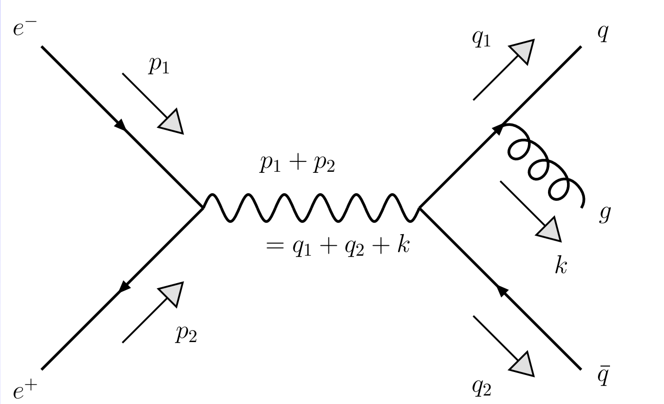
- The solution is to render the integrations finite in some way. This is made possible by the factorization properties of QCD matrix elements in soft and collinear limits

Factorization

- Examine the matrix element for $e^+e^- \to q\bar{q}g$ again:

$$\sum |\mathcal{M}|^2 = 8NC_F e^4 Q^2 g^2 \times$$
$$\frac{(p_1.q_1)^2 + (p_1.q_2)^2 + (p_2.q_1)^2 + (p_2.q_2)^2}{p_1.p_2 \; q_1.k \; q_2.k}$$



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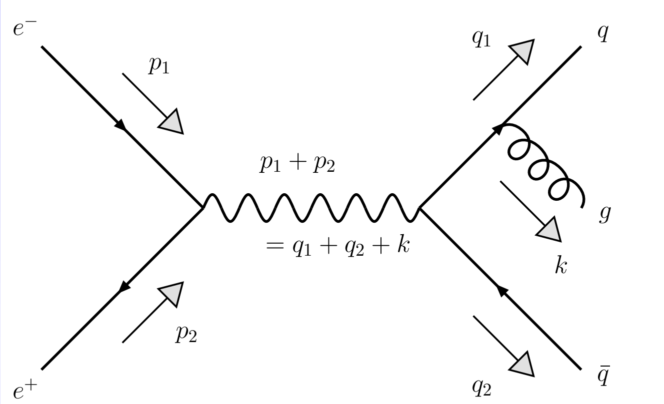

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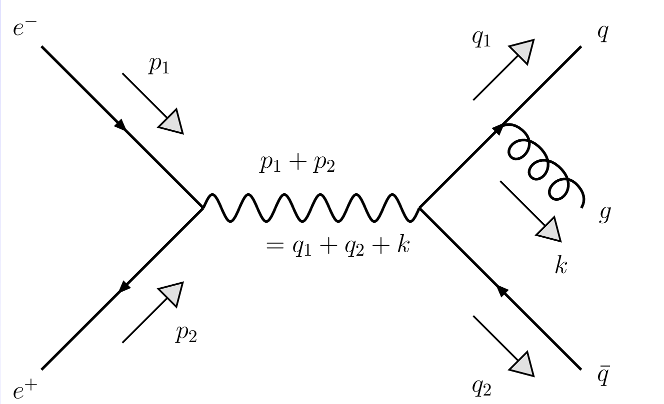

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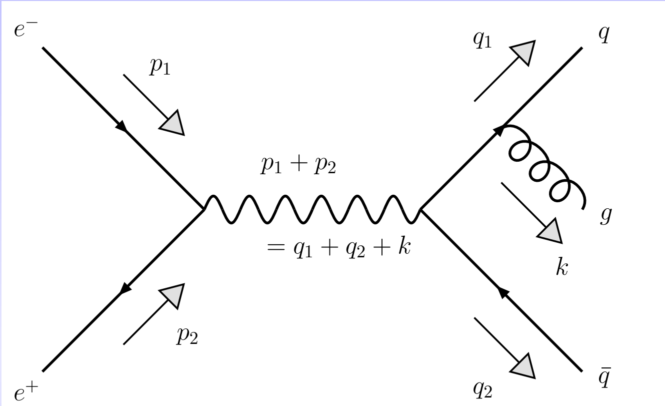

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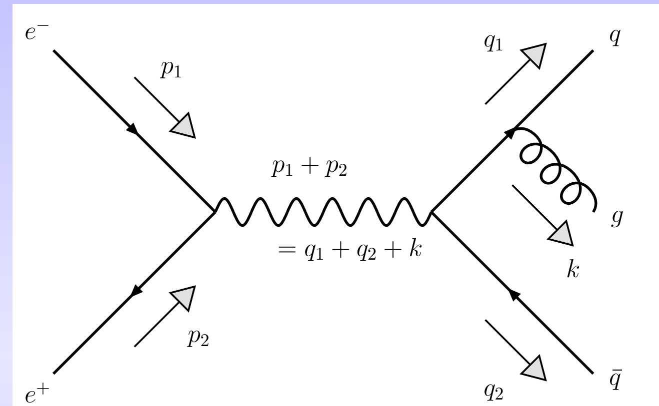

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- Equivalently,

$$\sum |\mathcal{M}|^2 \to C_F g^2 \frac{2p_1.p_2}{q_1.k \; q_2.k} |\mathcal{M}_{\mathrm{LO}}|^2$$

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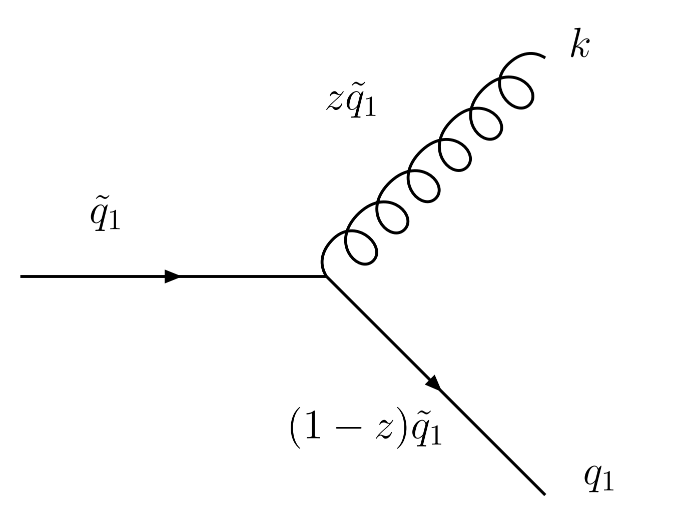

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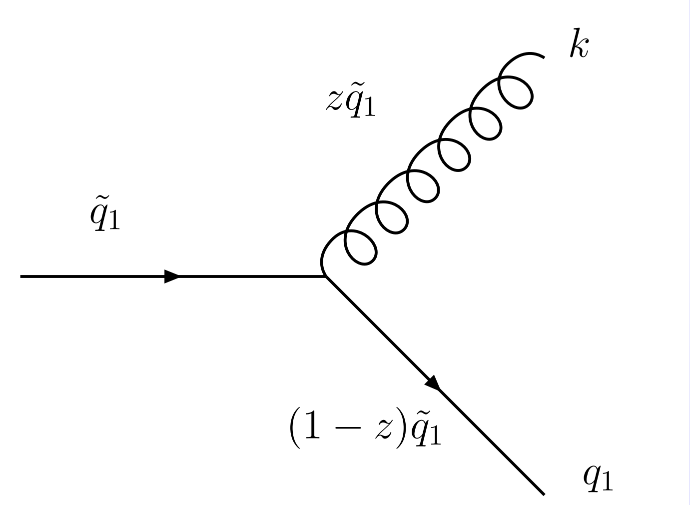

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- In this limit, we find that a similar (but more complicated) factorization occurs:

$$|\mathcal{M}_{q\bar{q}g}|^2 \longrightarrow 2g^2 C_F \, \frac{1}{2q_1.k} P_{qq}(z) \times |\mathcal{M}_{q\bar{q}}|^2$$

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- These functions are universal – they are sufficient to describe the soft and collinear behaviour of all QCD matrix elements

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- The integral should now be perfectly well-defined and suitable for numerical integration

Virtual terms

- We've dealt with the real diagrams, but what happens to the divergences in the virtual contribution?

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- By choosing a good parametrization, it is possible to factor the phase-space into the lowest order phase space multiplied by a region that represents the emission of an additional gluon:

$$dPS_{\rm LO+1} \quad\longrightarrow\quad dPS_{\rm LO}\;\times\; dPS_{\rm gluon}$$

Virtual result

- With this factorization, we can now integrate the counter-terms over this reduced phase-space:

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- What is remarkable is that the subtraction terms can be chosen such that they are both completely general (QCD factorization) and integrable (smart choices)

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- We will see examples of both of these techniques shortly

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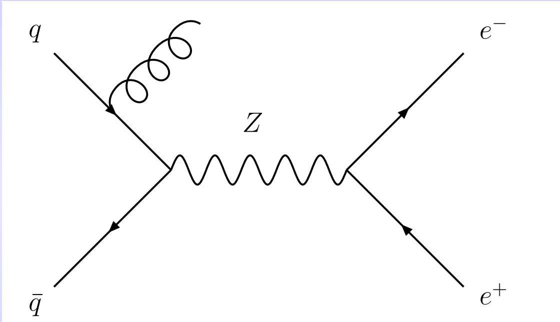

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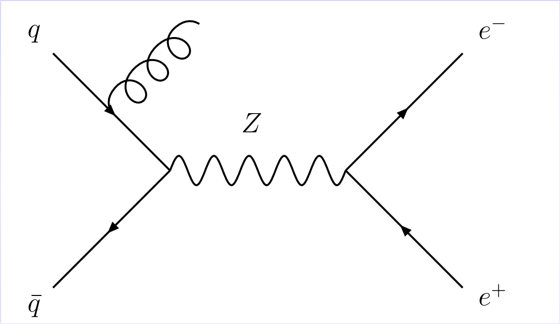

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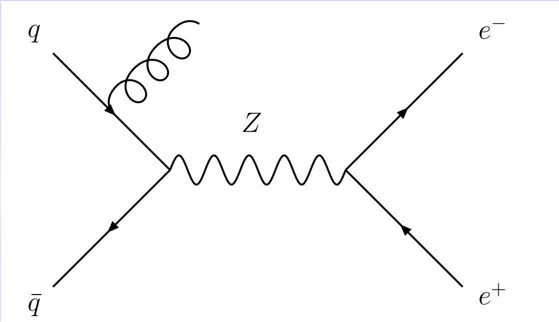

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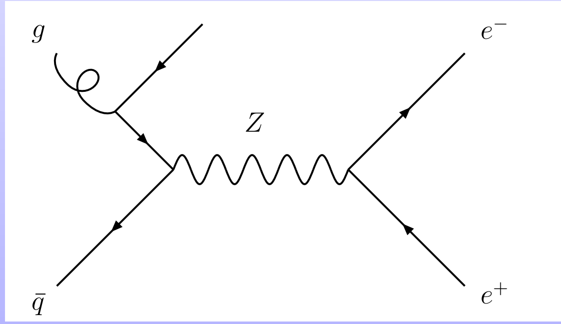

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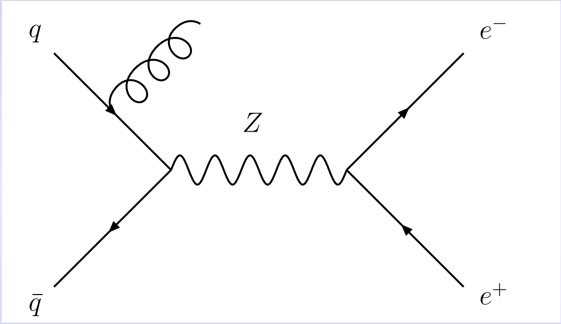

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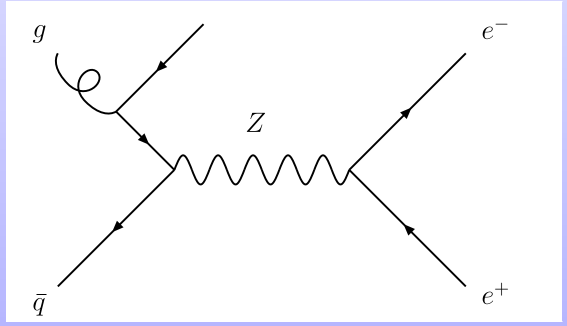

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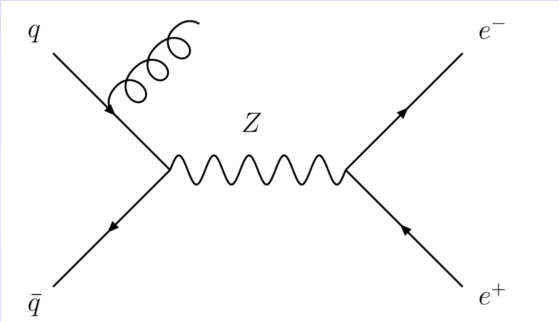

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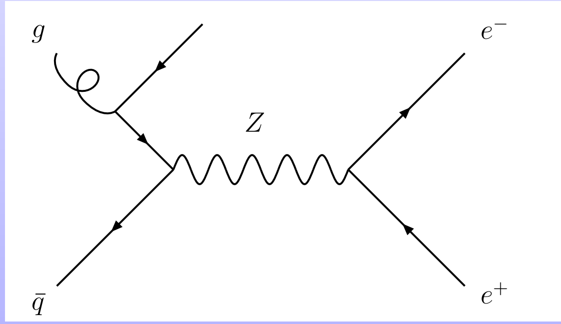

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- However, much of the machinery carries through as before

What do we gain from NLO?

We expect to see many benefits when performing a NLO calculation (examples coming soon):

- Less sensitivity to unphysical input scales
  - ⋆ first predictive normalization of observables at NLO
  - ⋆ more accurate estimates of backgrounds for new physics searches and (hopefully) interpretation
  - ⋆ confidence that cross-sections are under control for precision measurements
- More physics
  - ⋆ jet merging
  - ⋆ initial state radiation
  - ⋆ more parton fluxes
- It represents the first step for a plethora of other techniques
  - ⋆ matching with resummed calculations, NLO parton showers

NLO status

Given all the advantages of performing a NLO calculation, are the theoretical advances keeping up with the pace of progress in Run II at the Tevatron and construction at the LHC?

- What's the current state-of-the-art?
- Why are we lacking NLO predictions for many interesting processes that could be crucial to new physics discoveries in the near future?
  - traditional methods
  - where the difficulties lie
- What's being done about it?
  - promising new approaches
- Survey of available NLO tools for hadron colliders

An experimenter's wishlist

- Hadron collider cross-sections one would like to know at NLO

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}+\leq 3j$	$WW+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}+\leq 3j$	$WW+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$ZZ+b\bar{b}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{b}+\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$		$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}+\leq 3j$	$\gamma\gamma+b\bar{b}+\leq 3j$		
$\gamma+c\bar{c}+\leq 3j$	$\gamma\gamma+c\bar{c}+\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}+\leq 3j$		
	$WZ+c\bar{c}+\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

Theoretical status

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 2j$	$WW + \leq 0j$	$WWW + \leq 3j$	$t\bar{t} + \leq 0j$
$W + b\bar{b} + \leq 0j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 0j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 2j$	$ZZ + \leq 0j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 0j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 0j$
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	$W\gamma + \leq 0j$		
	$Z\gamma + \leq 0j$		

Slow progress

- Why has progress been so slow?

$e^+e^- \longrightarrow 3$ jets c. 1980

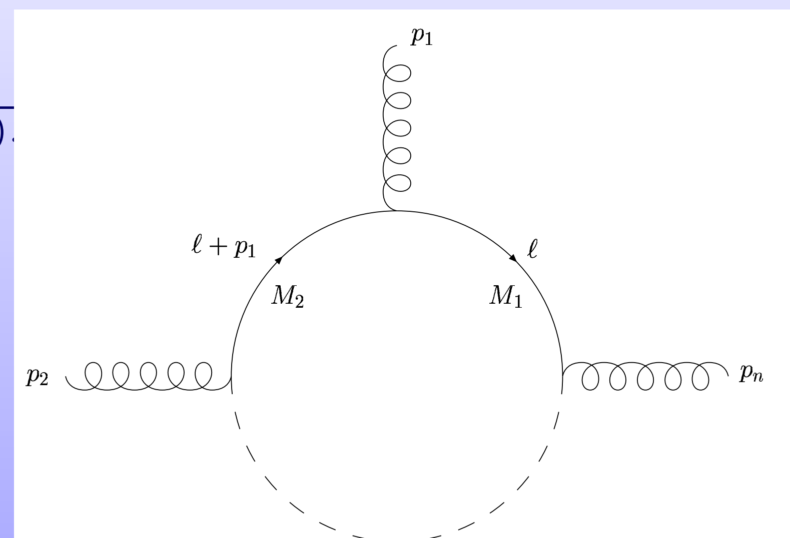
R. K. Ellis et al., 1981

$e^+e^- \longrightarrow 4$ jets c. 2000

Bern et al., Glover et al., 1996-7

- More particles $\rightarrow$ many scales $\rightarrow$ lengthy analytic expressions
- Integrals are complicated and process-specific:

$$\int d^{4-2\epsilon}\ell \, \frac{1}{(\ell^2 - M_1^2)((\ell+p_1)^2 - M_2^2)\ldots}$$



- different for:

$p_i^2 \neq 0$ W,Z,H

$M_i^2 \neq 0$ $t,b,\ldots$

Complications

- Fermions and non-Abelian couplings lead to more complicated tensor integrals:

$$\int d^{4-2\epsilon}\ell \, \frac{\ell^\mu}{(\ell^2 - M_1^2)((\ell + p_1)^2 - M_2^2)\ldots}$$

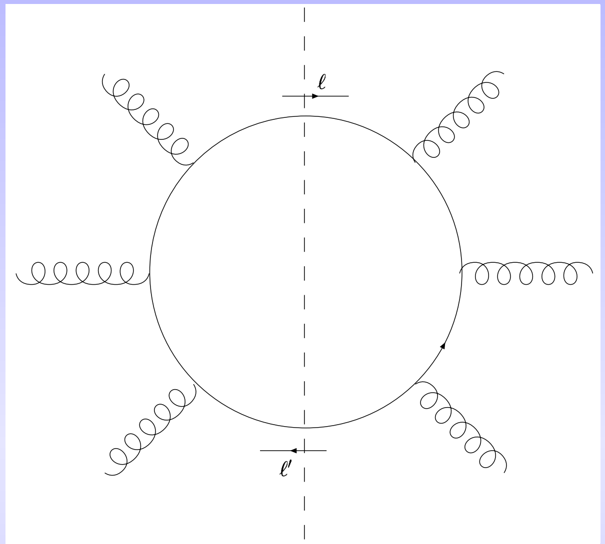
- Passarino-Veltman reduction in terms of scalar integrals:

$$\longrightarrow c_1 p_1^\mu + \ldots c_{n-1} p_{n-1}^\mu$$

  where the c_i are given by the solutions of $(n-1)$ equations

- This gives rise to the $(n-1) \times (n-1)$ Gram determinant, $\Delta = \det(2p_i \cdot p_j)$.
  - ⋆ large intermediate expressions
  - ⋆ spurious singularities

Unitarity technique



$\mathcal{M}_{tree}$ $\mathcal{M}'_{tree}$

$$= \int dPS(\ell, \ell')\, \mathcal{M}_{tree} \times \mathcal{M}'_{tree}$$

- Standard tree-level tricks can be used to simplify amplitudes, yielding compact results

  e.g. Dixon, hep-ph/9601359

- Rational functions of invariants cannot be obtained easily with this method
- Not easy to generalize and automate, simplification by hand

Numerical approach

- If all IR and UV singularities can be subtracted, perhaps loop integrals can be done numerically
- This method has many advantages:
  - a general solution for many processes, regardless of internal and external masses
  - extension to large final-state multiplicites limited only by CPU power
  - presence of masses in general should simplify the procedure (less singularities) rather than requiring much more work (cf. analytical approach)
- Several algorithms laid out, but no practical implementation so far

  Nagy and Soper, hep-ph/0308127

  Giele and Glover, hep-ph/0402152
- Exciting prospect for the future, but probably not until the LHC

NLOJET++

Author(s): Z. Nagy

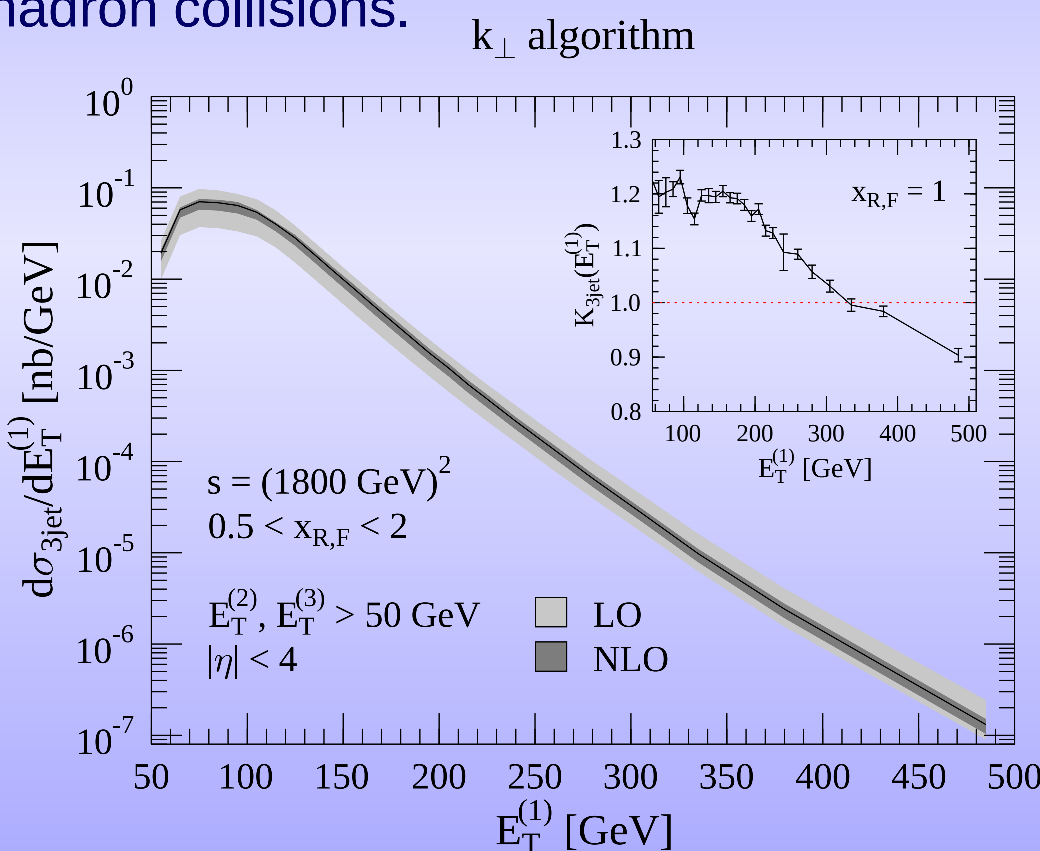
`http://www.ippp.dur.ac.uk/˜nagyz/nlo++.html`

Multi-purpose C++ library for calculating jet cross-sections in e^+e^- annihilation, DIS and hadron-hadron collisions.

$e^+e^- \longrightarrow \ \leq 4$ jets

$ep \longrightarrow \ (\leq 3+1)$ jets

$p\bar{p} \longrightarrow \ \leq 3$ jets



hep-ph/0110315

AYLEN/EMILIA

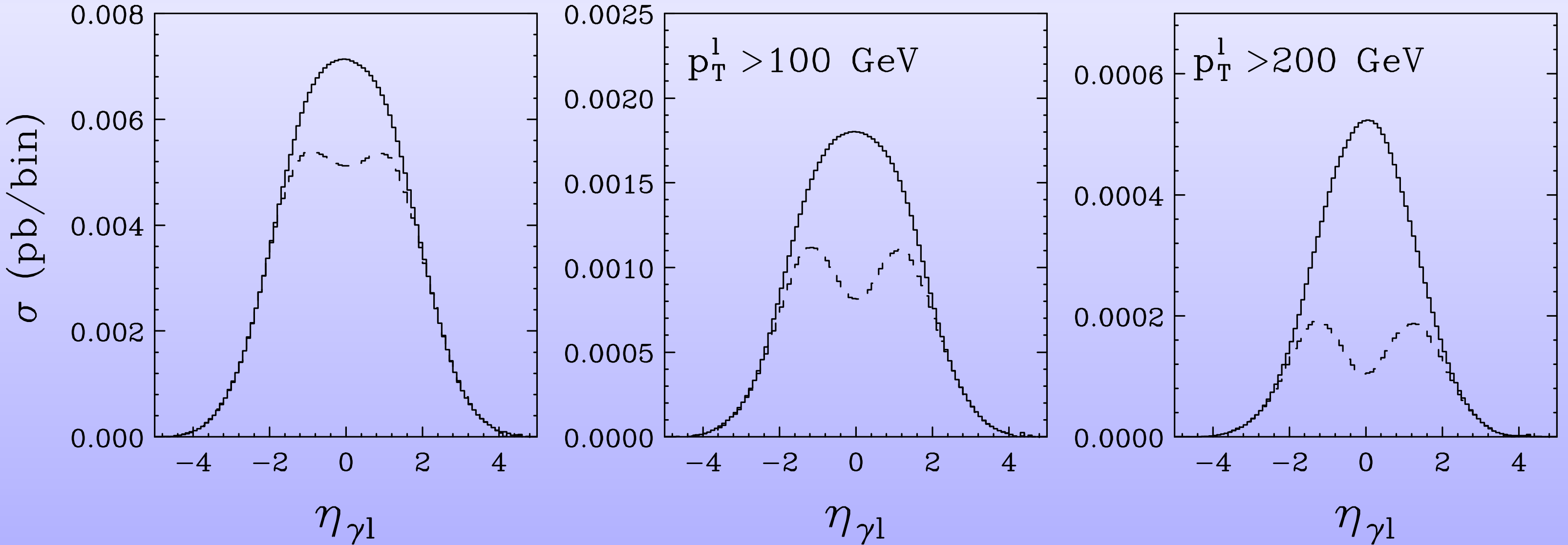
Author(s): L. Dixon, Z. Kunszt, A.Signer, D. de Florian

http://www.itp.phys.ethz.ch/staff/dflorian/codes.html

Fortran implementation of gauge boson pair production at hadron colliders, including full spin and decay angle correlations.

$p\bar{p} \longrightarrow VV'$ and $p\bar{p} \longrightarrow V\gamma$ with $V, V' = W, Z$

Anomalous triple gauge boson couplings at the LHC:



hep-ph/0002138

$\longrightarrow$ F. Olness

DIPHOX/EPHOX

Author(s): P. Aurenche, T.Binoth, M. Fontannaz, J. Ph. Guillet, G. Heinrich, E. Pilon, M. Werlen

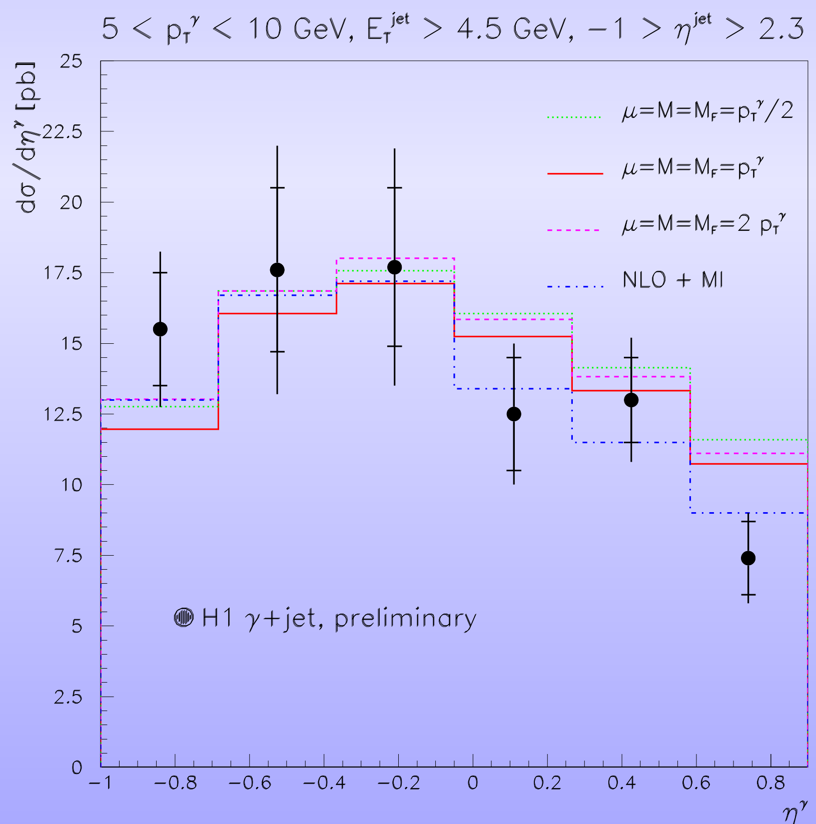
`http://wwwlapp.in2p3.fr/lapth/PHOX_FAMILY/main.html`

Fortran code to compute processes involving photons, hadrons and jets in DIS and hadron colliders.

$$p\bar{p} \longrightarrow \gamma + \leq 1 \text{ jet}$$

$$p\bar{p} \longrightarrow \gamma\gamma$$

$$\gamma p \longrightarrow \gamma + \text{ jet}$$



Preliminary H1 data, `hep-ph/0312070`.

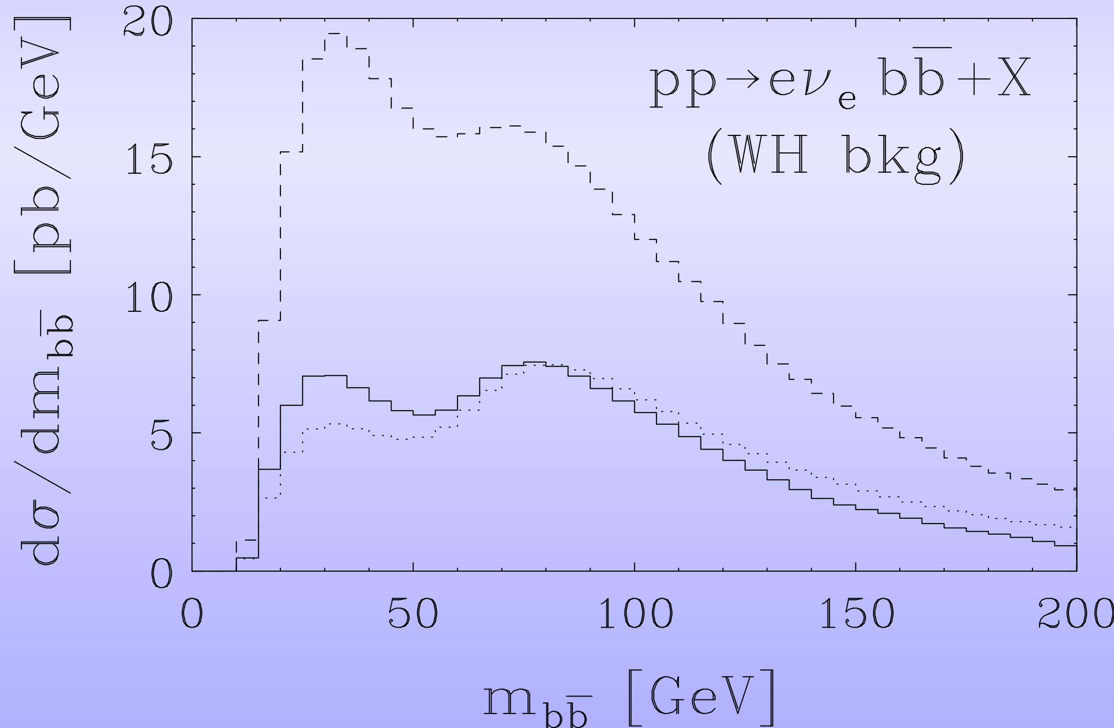
MCFM

Author(s): JC, R. K. Ellis
http://mcfm.fnal.gov
Fortran package for calculating a number of processes involving vector bosons, Higgs, jets and heavy quarks at hadron colliders.

$p\bar{p} \longrightarrow V+ \leq 2$ jets

$p\bar{p} \longrightarrow V + b\bar{b}$

with $V = W, Z$.

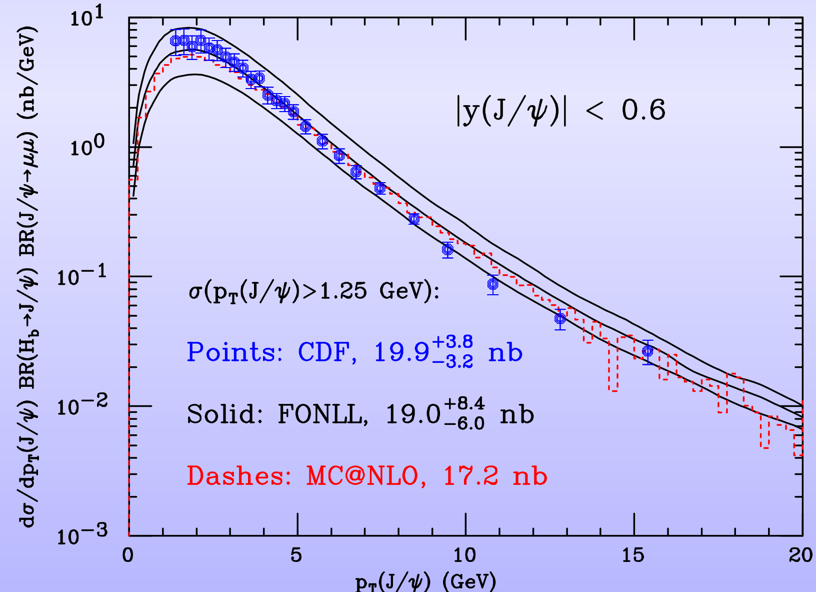


hep-ph/0308195

Heavy quark production

Author(s): M. L. Mangano, P. Nason and G. Ridolfi
`http://www.ge.infn.it/~ridolfi/hvqlibx.tgz`
Fortran code for the calculation of heavy quark cross-sections and distributions in a fully differential manner

- Based on the more inclusive calculations of Dawson et al, Beenakker et al.
- Does not include multiple gluon radiation, $\log(p_T/m_b)$ (FONLL)
  Cacciari et al., hep-ph/9803400
- These are the same matrix elements that are incorporated into MC@NLO
  Frixione et al., hep-ph/0305252



hep-ph/0312132

$\longrightarrow$ R. K. Ellis

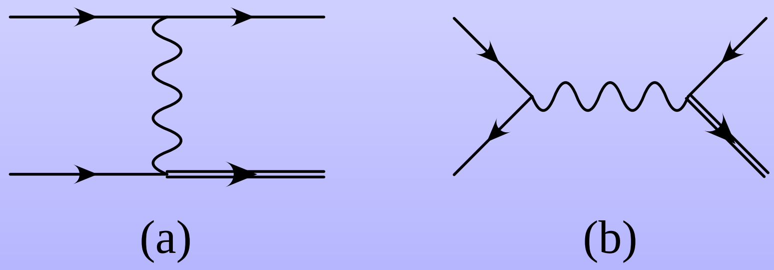
Single top production

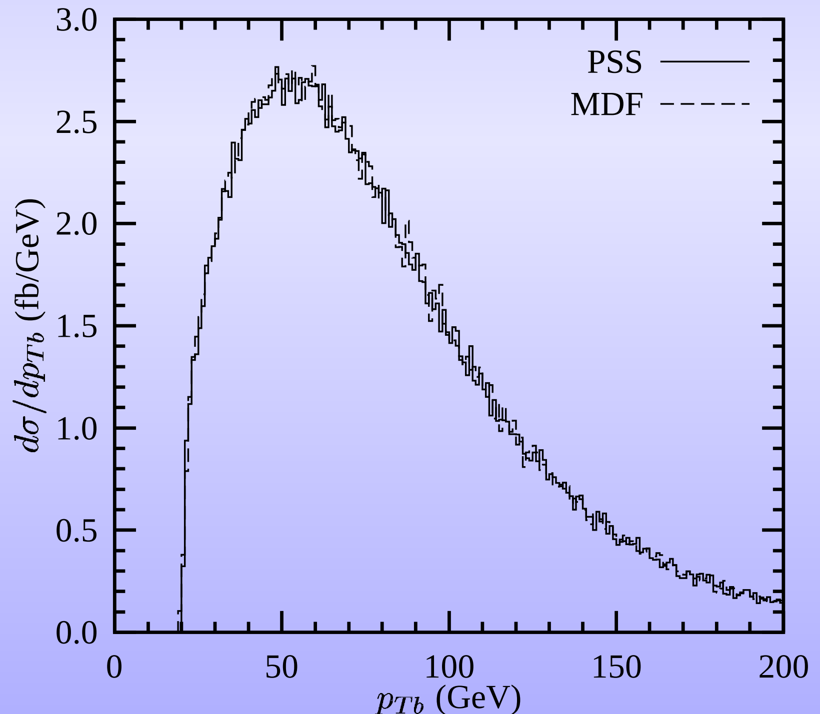
Author(s): B. W. Harris, E. Laenen, L. Phaf, Z. Sullivan, S. Weinzierl
(No public code released)

Fully differential calculation of single top production in hadron-hadron collisions, via both channels:

(a) $u + b \longrightarrow t + d$

(b) $u + \bar{d} \longrightarrow t + \bar{b}$





hep-ph/0207055

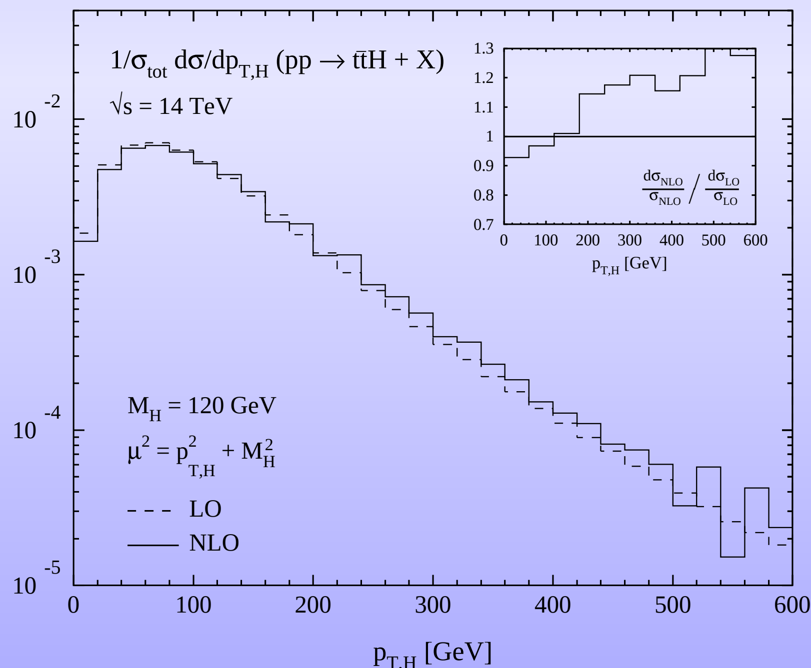
$\longrightarrow$ T. Tait

Higgs + $Q\bar{Q}$

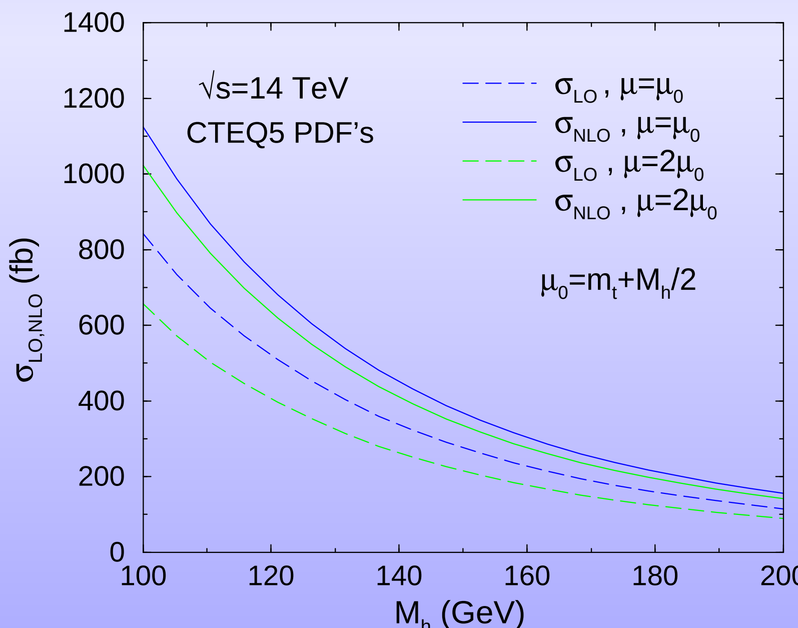
Author(s): S. Dawson, C. B. Jackson, L. H. Orr, L. Reina, D. Wackeroth; W. Beenakker, S. Dittmaier, M. Kramer, B.Plumper, M. Spira, P. Zerwas
(No public code released)

Associated production of a Higgs and a pair of heavy quarks,

$$p\bar{p} \longrightarrow Q\bar{Q}H, \quad \text{with } Q = t, b.$$



hep-ph/0211352



hep-ph/0311216

$\longrightarrow$ B. Kilgore

HEPCODE database

- A new initiative to maintain a list of available Monte Carlo codes, including lowest order, NLO and resummed predictions
- Eventual aim is to produce a searchable database

`http://www.ippp.dur.ac.uk/~wjs/HEPCODE/`

HEPCODE PROGRAMME LISTING

The idea of making a comprehensive database of programmes for cross section calculations and event simulations arose out of a discussion at the Collider Physics Conference at the KITP, Santa Barbara in January 2004. The database will eventually be integrated into the HEPDATA databases in Durham, and will incorporate a "search" facility that will enable users to identify a set of available programmes simply by entering the details of a particular scattering process. In the meantime, we need to build up a comprehensive list of all available codes. The emphasis so far is on hadron-collider processes, but it is hoped to eventually include also a comprehensive list for other colliders.

Comments on the list below (for example, if your programme is listed but the information is incomplete/incorrect) and particularly suggestions for new entries are very welcome and should be sent to James Stirling (IPPP, Durham) at w.j.stirling@durham.ac.uk, using the automated submission tool.

(Thanks to: John Campbell, Guenther Dissertori, Thomas Gehrmann, Bill Kilgore, Adrian Signer)

Key

- ee, ep, pp are used as shorthand for electron-positron, lepton-hadron, and proton - (anti)proton collisions respectively
- V = W or Z, and sometimes also a Drell-Yan virtual photon, g = real photon, l = lepton, H = Higgs boson
- j = light (u,d,s,c?) quark or gluon jet; Q = generic heavy (c?,b,t) quark
- TL = tree level; PS = parton shower; NLO = NLO QCD, NNLO = NNLO QCD; NLOEW = NLO electroweak, RS=resummed
- F = Fortran, C = C++

Name/ description	processes	order	code?		authors	comments
VECBOS	pp V + <=4j	TL	yes	F	W. Giele	
ALPGEN	pp V + QQbar + <=4j V + <=6j V + c + <=5j nV + mH + <=3j QQbar + <=6j QQbar + Q'Q'bar + <=4j QQbar + H + <=4j <=6j ng + mj, n+m<=8, m<=6	TL	yes	F	M. Mangano M. Moretti F. Piccinini R. Pittau A. Polosa	a collection of codes for the generation of multi-parton processes in hadronic collisions based on the Alpha matrix element generator
MADEVENT		TL	yes	F	T. Stelzer F. Maltoni	combines MADGRAPH matrix element calculations with phase space integration
HELAC		TL	yes	F	C. Papadopoulos	
AMEGIC++		TL	yes	C	F. Krauss	
GRACE		TL	yes	F		
GR@PPA	pp bbbar + bbbar V + <=3j VV' ttbar W + <=2g	TL	yes	F	S. Tsuno S. Shimma J. Fujimoto T. Ishikawa Y. Kurihara S. Odaka	an extension of the GRACE system to hadron collider processes; includes full decays of vector bosons and top quarks; can be embedded in PYTHIA and HERWIG
COMPHEP		TL	yes	F	A. Pukhov E. Boos M. Dubinin V. Edneral V. Ilyin D. Kovalenko A. Kryukov V. Savrin S. Shichanin A. Semenov	
AcerMC	pp ...	TL	yes	F	B. Kersevan E. Richter-Was	generates a variety of Standard Model background

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S. Frixione and B. R. Webber, hep-ph/0402116

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  S. Frixione and B. R. Webber, hep-ph/0402116
- Expect more progress in this direction in the future

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Thoughts to leave with ...

- NLO tools are an invaluable aid to experimental studies now and will continue to be so in the future
- It's important to have at least a basic grasp of the underlying theory, if only to appreciate the feasibility of a desired calculation
  - ⋆ Even though PYTHIA and HERWIG have been the simulation tools of choice in the past, it's likely that the next generation of programs will be based on a NLO core
- There are many programs available for making NLO predictions at the Tevatron and the LHC, in a variety of forms:
  - ⋆ author-controlled
    single top, $H + Q\bar{Q}$
  - ⋆ single class of processes
    $V\gamma$, $Q\bar{Q}$
  - ⋆ generic programs
    NLOJET++, PHOX-family, MCFM